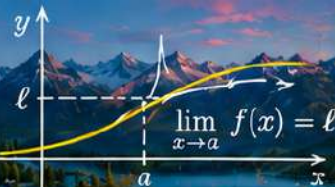


Limites de fonctions et dérivation

Fiche d'exercices n°2 – Corrigé

Techniques de calcul et formes indéterminées

$$\begin{array}{l} \frac{0}{0} \quad \frac{+\infty}{\infty} \\ \infty - \infty \\ 0 \times \infty \quad 1^\infty \end{array}$$



1 Formes indéterminées 0/0 – Factorisations classiques

Exercice 1 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)} \\ \quad (\text{lim } (x+1)) = 2 \\ \text{b) } \lim_{x \rightarrow 2} \frac{x^2 - 4}{x + 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x+2)} \\ \quad = \lim_{x \rightarrow 2} (x-2) = 0 \\ \text{c) } \lim_{x \rightarrow 0} \frac{x^2 + 3x}{x} = \lim_{x \rightarrow 0} \frac{x(x+3)}{x} \\ \quad = \lim_{x \rightarrow 0} (x+3) = 3 \\ \text{d) } \lim_{x \rightarrow 1} \frac{x^3 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x(x-1)(x+1)}{(x-1)(x+1)} \\ \quad = \lim_{x \rightarrow 1} x = 1 \end{array}$$

Exercice 2 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x-2)}{(x-1)} \\ \quad = \lim_{x \rightarrow 1} (x-2) = -1 \\ \text{b) } \lim_{x \rightarrow 3} \frac{2x^2 + 5x - 3}{x + 3} = \lim_{x \rightarrow 3} \frac{(2x-1)(x+3)}{(x+3)} \\ \quad = \lim_{x \rightarrow 3} (2x-1) = 5 \\ \text{c) } \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)} \\ \quad = \lim_{x \rightarrow 2} (x^2 + 2x + 4) = 12 \\ \text{d) } \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)} \\ \quad = \lim_{x \rightarrow 3} (x+3) = 6 \end{array}$$

Exercice 3 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} \\ \quad = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2} \\ \text{b) } \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{4} \\ \text{c) } \lim_{x \rightarrow 0} \frac{\sqrt{x} + 1 - 1}{x} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x} + 1} = 1 \\ \text{d) } \lim_{x \rightarrow 0} \frac{\sqrt{2x+3} - \sqrt{3}}{x} = \frac{2}{\sqrt{2x+3} + \sqrt{3}} \\ \quad = \frac{1}{\sqrt{3}} \end{array}$$

Exercice 4 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (\text{limite remarquable}) \\ \text{b) } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \quad (1 - \cos x \sim \frac{x^2}{2}) \\ \text{c) } \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = 0 \quad (x - \sin x \sim \frac{x^3}{6}) \\ \text{d) } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad (\tan x \sim x) \end{array}$$

Exercice 5 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \quad (e^x - 1 \sim x) \\ \text{b) } \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = 2 \quad (e^{2x} - 1 \sim 2x) \\ \text{c) } \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2} \quad (e^x - 1 - x \sim \frac{x^2}{2}) \\ \text{d) } \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1 \quad (\ln(1+x) \sim x) \end{array}$$

2 Formes indéterminées ∞/∞ et quotient de polynômes

Exercice 6 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow +\infty} \frac{3x^2 + 2x - 1}{x^2 - 5} = 3 \\ \text{b) } \lim_{x \rightarrow +\infty} \frac{-2x^3 + 5x}{x^3 + 1} = -2 \\ \text{c) } \lim_{x \rightarrow +\infty} \frac{4x^4 - x^2 + 1}{2x^4 + 3x^2 - 7} = 2 \\ \text{d) } \lim_{x \rightarrow -\infty} \frac{x^3 - 6x^2 + x}{-x^3 + 4x + 5} = -1 \end{array}$$

Exercice 7 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow +\infty} \frac{2x^2 - 3x + 1}{x^2} = 2 \\ \text{b) } \lim_{x \rightarrow +\infty} \frac{x^3 + 2x}{x^2 - x + 1} = +\infty \\ \text{c) } \lim_{x \rightarrow +\infty} \frac{5x^4 - 3x + 7}{2x^2 + 1} = +\infty \\ \text{d) } \lim_{x \rightarrow +\infty} \frac{-3x^3 + 2x^2}{4x^3 - x + 1} = -\frac{3}{4} \end{array}$$

Exercice 8 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow +\infty} \frac{2x+1}{x\sqrt{x}} = 0 \quad (\simeq \frac{2}{\sqrt{x}}) \\ \text{b) } \lim_{x \rightarrow +\infty} \frac{\sqrt{x}+3}{x-1} = 0 \quad (\simeq \frac{1}{\sqrt{x}}) \\ \text{c) } \lim_{x \rightarrow +\infty} \frac{x^2 + \sqrt{x}}{x^3 - 2} = 0 \quad (\simeq \frac{1}{x}) \\ \text{d) } \lim_{x \rightarrow +\infty} \frac{3x-1}{\sqrt{x}+2} = +\infty \quad (\simeq 3\sqrt{x}) \end{array}$$

Exercice 9 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow +\infty} \frac{2x^2 + 1}{x^2 - 3x + 2} = 2 \\ \text{b) } \lim_{x \rightarrow +\infty} \frac{x^3 + x^2 + 1}{x^3 - 5x + 4} = 1 \\ \text{c) } \lim_{x \rightarrow -\infty} \frac{4x^4 - 2x^2 + x}{-3x^4 + 5x^2 - 1} = -\frac{4}{3} \\ \text{d) } \lim_{x \rightarrow +\infty} \frac{x^2 - 7x + 6}{x^2 + 2x - 8} = 1 \end{array}$$

3 Formes indéterminées particulières et rationalisation

Exercice 10 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^2 - 1} = \frac{1}{2} \quad \text{b) } \lim_{x \rightarrow 2} \frac{\sqrt{x} - 2}{x - 2} = \frac{1}{4} \\ \text{c) } \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} = 4 \quad \text{d) } \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x} + 1} = 0 \end{array}$$

Exercice 11 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + 1 - \sqrt{x}}{1/x} = 1 \quad \text{b) } \lim_{x \rightarrow +\infty} (\sqrt{x} - \sqrt{x-1}) = 0 \\ \quad (\text{différence de deux quantités voisines}) \\ \text{c) } \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - x) = \frac{1}{2} \quad \text{d) } \lim_{x \rightarrow +\infty} (x - \sqrt{x^2 + 1}) = -\frac{1}{2} \\ \quad (\text{rationalisation}) \end{array}$$

Exercice 12 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \quad \text{b) } \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1}{2} \\ \text{c) } \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2} \quad \text{d) } \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x^2} = -\frac{1}{2} \end{array}$$

Exercice 13 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x} = 2 \quad (e^x - e^{-x} \sim 2x) \\ \text{b) } \lim_{x \rightarrow 0} \frac{\sin 2x}{x} = 2 \quad (\sin 2x \sim 2x) \\ \text{c) } \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2 \quad (1 - \cos 2x \sim 2x^2) \\ \text{d) } \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x^2} = 3 \quad (\text{quotient remarquable}) \end{array}$$

4 Choix de la technique et exercice bilan

Exercice 14 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 1} \frac{x - 1 - \ln x}{x - 1} = 0 \quad (\ln x = (x-1) + o(x-1)) \\ \text{b) } \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{\ln(1+x)} = \frac{1}{2} \quad (\sqrt{1+x} - 1 \sim \frac{x}{2}, \ln(1+x) \sim x) \\ \text{c) } \lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{x^2}{2}}{x^3} = \frac{1}{6} \quad (e^x \text{ développement limité}) \\ \text{d) } \lim_{x \rightarrow 2} \frac{x^3 - 6x + 8}{x^2 - 4} = 3 \quad (\text{factorisation}) \end{array}$$

Exercice 15 – Corrigé

$$\begin{array}{l} \text{a) } \lim_{x \rightarrow 0} \frac{e^x - 1}{\sqrt{x+1} - 1} = 2 \quad \text{c) } \lim_{x \rightarrow 0} x \ln x = 0 \quad \lim_{x \rightarrow 0} x^2 \ln x = 0 \\ \quad (x \ln x \rightarrow 0 \text{ lorsque } x \rightarrow 0^+) \\ \text{b) } \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2} = -\frac{1}{8} \quad (\text{développement limité}) \\ \text{d) } \lim_{x \rightarrow +\infty} \frac{x^3 - 6x + 8}{x^2 - 4} = +\infty \quad (\text{comportement dominant } x) \end{array}$$

